

An Equation For Charge Decay Valid in Both Conductors and Insulators

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Abstract Gauss' law and the equation of continuity must be satisfied in all materials be they solids, liquids or gases. Most materials are classified as simple materials; i.e., their electrical properties are linear, isotropic and homogeneous. Charge transport in these simple materials should be described by a constitutive equation known as Ohm's law. When Ohm's law is combined with Gauss' law and the equation of continuity, a differential equation for volume charge density relaxation results. The usual solution to this equation shows that charge decays exponentially with a relaxation time given by the material's permittivity divided by its electrical conductivity. Experiments show that good conductors follow this exponential decay but that poor conductors (insulators) tend to follow a decay that initially is more hyperbolic than exponential. This suggests that either Ohm's law is not valid for insulator materials or that a deeper understanding of Ohm's law is needed to explain charge decay in these less than good conductors. This paper examines the latter approach and shows that, when all the free charges within a simple material are taken into account, a new closed-form equation for unipolar charge decay can be derived which is valid for any simple material be it classified as a very good conductor, or a very poor conductor or anywhere in between. For good conductors the equation reduces to the standard *exponential law* of decay. For very poor conductors the equation reduces to the Vellenga-Klinkenberg *modified hyperbolic law* with the initial decay producing the characteristic Bustin *hyperbolic law* of decay. Explicit definitions for a good conductor and a good insulator are obtained and are used to define the range where explicit deviations from both of these hyperbolic laws occur.

Introduction

Electrostatics is the study of charges including their effects and how they affect the local environment. In other words, electrostatics examines the behavior of charges in various material media (solids, liquids and gases). In the absence of magnetic influences, the relevant electrical properties of a material are its electrical conductivity σ and its permittivity ϵ given by $\epsilon = \epsilon_0 \epsilon_r$, where $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is the permittivity of free space and ϵ_r is the relative permittivity or dielectric constant of the material.

When excessive charge builds up in a material, it can often have dire consequences such as initiation of a spark that results in a fire, explosion or other damage to the local environment. For example, in every operation from the simple (e.g., walking) to the complicated (e.g., semiconductor manufacture) there is an operation time (e.g., walking step, ion-sputtering time, etc.) during which

a material (e.g., sole of shoe, semiconductor chip) can become charged. If the rate at which charge is deposited is greater than the rate at which charge can decay, a charge build-up on or within the material can occur. As a result, it is important to understand charge decay in sufficient detail so as to be able to predict if a material constitutes an electrostatic risk in a particular operation.

In order to gain a more fundamental insight into charge decay, this paper first reviews the basic concept of applying Gauss' law and the equation of continuity to simple materials; i.e., materials that conform to Ohm's law. Then a more in-depth review of the physics which leads to Ohm's law is presented including the concept of electrical conductivity. This is followed by a more in-depth review of the partial differential equation that results when Ohm's law is combined with Gauss' law and the equation of continuity. After defining the concept of a material (solid, liquid or gas; conductor or insulator) as being electrically neutral in the intrinsic state, a unipolar charge species is introduced into a simple material. It is shown that this charge temporarily changes the overall electrical conductivity, a point either missed or ignored in past charge decay derivations. Once it is recognized that the conductivity has changed due to the charge, a closed form equation that describes the decay of the charge is developed. The new equation reduces to the single exponential decay for a material classified as a good conductor. For a good insulator the new equation reduces to the known "hyperbolic decay law" during the initial decay time.

Traditional Charge Decay Development

Most materials can be classified as "simple" materials and this paper deals only with such materials which are defined as isotropic, linear and homogeneous [1]. Isotropic means that the polarization vector points in the direction of the electric field vector \mathbf{E} . Linear means the electric susceptibility is a scalar constant so the polarization vector and electric field vector are linearly related, and hence, the material's permittivity $\epsilon \neq \epsilon(\mathbf{E})$ is not a function of the electric field. Homogeneous means no spatial gradients and hence $\nabla\epsilon = 0$ and $\nabla\sigma = 0$. Within a material Gauss' law gives the relationship between the electric field \mathbf{E} at a point and the net charge density ρ at that point as [1, 2]

$$\nabla \cdot \epsilon \mathbf{E} = \rho \quad (1)$$

where ∇ is the usual "del" operator ([1]; pp. 39-43). The general continuity equation includes source terms (e.g., rate of charge generation) and sinks terms (e.g., rate of charge recombination) [3, 4]. At times these rate terms must be included in the equation of continuity. However, if sources are equal to sinks at all points or if these sources and sinks do not exist, the net charge flux or current density vector must satisfy the simplified (source and sink free) equation of continuity [1, 2]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0. \quad (2)$$

A simple material follows Ohm's law given by [1, 2]

$$\mathbf{J} = \sigma \mathbf{E}. \quad (3)$$

For a simple material that follows Ohm's law, combining (2) and (3) gives

$$\frac{\partial \rho}{\partial t} + \sigma \nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla \sigma = 0. \quad (4)$$

As noted above for a simple material $\nabla \sigma = 0$ and $\nabla \epsilon = 0$ so (1) and (4) yield

$$\frac{\partial \rho}{\partial t} + \sigma \frac{\rho}{\epsilon} = 0. \quad (5)$$

The traditional method to solve (5) is simply to state when charge density ρ is added that σ and ϵ are still constants and define the electrical relaxation time of the conductor as

$$\tau_c = \frac{\epsilon}{\sigma} \quad (6)$$

and integrate (5) from time $t = 0$ when the charge density is ρ_0 to general time t obtaining

$$\rho(t) = \rho_0 \exp(-t/\tau_c). \quad (7)$$

If a unipolar charge is placed in a material, the coulomb force will push the charges away from each other. Eventually all charges will arrive at the surface so (7) indicates the net charge ρ at all points inside the material will decay to zero in approximately $5\tau_c$. The electrical conductivity of copper is 5.8×10^7 S/m and the dielectric constant ϵ_r is 1 giving $\tau_c = 1.53 \times 10^{-19}$ second, whereas for fused quartz $\sigma = 10^{-17}$ S/m and $\epsilon_r = 5$ giving $\tau_c = 51.2$ days; so, in many applications copper is considered a good conductor moving any injected charge to the surface of the conductor within 10^{-18} second whereas quartz is considered an insulator suggesting that for the first several hours any charge introduced in quartz will behave as if it had remained essentially wherever it was placed [2].

In practice, it is found that metals and other good conductors follow (7) but that for insulator materials the initial decay appears hyperbolic with a decay rate dependent on the initial charge present. As a result (7) is clearly not valid for both conductors and insulators. Later it will be argued that although (5) is the proper partial differential equation to describe the relaxation of the net charge with time, a more thorough investigation of Ohm's law shows that when charge is added to a material $\sigma = \sigma(t)$ and, hence, the traditional method of solving (5) described above to obtain (7) is not actually valid.

Ohm's Law Revisited

Today there is sufficient evidence to indicate matter is made up of *neutral* atoms and groups of atoms called molecules. Atoms are believed to be made up of a nucleus containing protons and neutrons with the nucleus surrounded by electrons. Neutral atoms have equal numbers of protons and electrons. An electron has a negative charge measured at -1.6×10^{-19} coulomb (C) and a proton has a positive charge of $+1.6 \times 10^{-19}$ C. It is very difficult to remove a proton from an atom, but an electron can be more easily removed. As a result, positive charges occur in matter when electrons are removed from neutral atoms; and negative

charges occur when electrons are added (attached) to neutral atoms or remain free. This information and especially the fact that the intrinsic composition of a material is made up of *neutral* atoms and *neutral* molecules is important in understanding the nature of Ohm's law.

The general charge flux equation for a single species of free charge is well known and given by ([3]; see their Eq. 18)

$$\mathbf{J}_i = \rho_i \mathbf{v}_{d0} + \sigma_i \mathbf{E} - D_i \nabla \rho_i - G_i \rho_i \nabla T. \quad (8)$$

Equation (8), which is also referred to as a constitutive law [5] can be obtained the traditional way by studying individually each transport process that can occur in a volume element at a point and then adding all these processes together [6] or by applying the contiguous-collision-averaging method to a point in space [7]. The nomenclature used here is that given in [7] in which the sign of charge s_i is explicit so that the charge density is [7]

$$\rho_i = s_i q_i n_i \quad (9)$$

where n_i is the number density of the charge species having a charge of magnitude q_i where $q_i = Z_i q_0$ with $q_0 = +1.6 \times 10^{-19}$ coulomb being the fundamental unit of charge and Z_i being the number of fundamental units. The term s_i is the sign of the charge where $s_i = 1$ for positive charge and $s_i = -1$ for negative charge. As a result $q_i n_i$ is always a positive quantity and the sign of ρ_i is determined by the sign of s_i . The introduction of the term s_i allows the mobility constant b_i to always be introduced as a positive quantity where the velocity of the charge is given by $\mathbf{v}_i = s_i b_i \mathbf{E}$. The conductivity of the i^{th} species is [7]

$$\sigma_i = s_i^2 q_i n_i b_i \quad (10)$$

The diffusion constant of the i^{th} charged species is D_i and the thermophoresis coefficient is G_i , but these need not be considered further here as this paper is restricted to homogeneous materials ($\nabla \rho_i = 0$) at uniform temperature ($\nabla T = 0$). The drift velocity \mathbf{v}_{d0} in (8) of the material can often be important in liquids and gases and is zero for a stationary solid, liquid or gas.

Usually more than one species of charge exists in a material. For example, in a neutral gas ionization removes electrons from neutral molecules producing free electrons as one species and positive ions as another species. The electrons can then attach to neutral molecules and produce negative ions as a third species. A similar result occurs in liquids due to dissociation. In solids there can be many (conductor), some (semiconductor) or few (insulator) free electrons. Electrons freed from atoms in solids also produce positive charged atoms, but these atoms cannot physically move. However, by considering these charged atoms as holes, when an electron moves from its neutral atom to a previously charged atom (hole) the original hole disappears but a new hole is created. This effective hole motion is treated as a moving charged species [8]. In general the total charge flux is thus given by the sum of all the charge fluxes [7, 9]

$$\mathbf{J} = \sum_i \mathbf{J}_i. \quad (11)$$

Consequently, (8) and (11) combine to define the total charge flux as

$$\mathbf{J} = \sum_i \rho_i \mathbf{v}_{d0} + \sum_i \sigma_i \mathbf{E} - \sum_i D_i \nabla \rho_i - \sum_i G_i \rho_i \nabla T. \quad (12)$$

In (12) the total or net volume charge density in the material at any point is given by [7]

$$\rho = \sum_i \rho_i. \quad (13)$$

Similarly the total electrical conductivity of the material at any point is [7]

$$\sigma = \sum_i \sigma_i. \quad (14)$$

With (13) and (14) it is possible to write (12) as [7]

$$\mathbf{J} = \rho \mathbf{v}_{d0} + \sigma \mathbf{E} - \sum_i D_i \nabla \rho_i - \left(\sum_i G_i \rho_i \right) \nabla T. \quad (15)$$

As pointed out in [7] many materials have no convective drift $\mathbf{v}_{d0} = 0$, are homogeneous $\nabla \rho_i = 0$ (i.e., are a simple material) and have a uniform temperature $\nabla T = 0$. For these conditions (15) reduces to Ohm's law as given by (3). Note that nothing in what has so far been presented requires the total electrical conductivity (14) to be a constant with time. However, as Maxwell pointed out, Ohm's law would have little scientific value if a specific property of the material (i.e., the intrinsic conductivity) could not be defined [10]. This intrinsic electrical conductivity, which is not a function of time, will be clearly defined in the next section.

Consequences of Ohm's Law

In order to understand the concept of introducing free charge into a material the concept of intrinsic conductivity must first be understood. The intrinsic conductivity is the electrical conductivity of a material before the introduction of external free charges. This can be best understood by examining the implications of (13) and (14) regarding a material's ρ and σ before external charge is introduced (Case 1). Then the complication of introducing a single species of free charge (Case 2) can be easily followed.

Case 1: (Charge neutrality; intrinsic conductivity)

Because all atoms have equal amounts of positive and negative charge and because all materials are made up of atoms, all materials in their equilibrium or intrinsic state will be neutral. Therefore, the intrinsic net charge must consist of equal numbers of positive and negative charge; consequently, the intrinsic net charge must be zero. This analysis can be easily extended to any number of *sets* of different charge species but consider here the simplest situation were

only one set of each charge (i.e., two charged species, one positive and the other negative) occurs. When no other charges are present except those from the material's atoms and molecules, the equilibrium density is from (9) and (13)

$$\rho(eq) \rightarrow \rho_m = \rho_+ + \rho_- = s_+ q_+ n_+ + s_- q_- n_- = s_+ (q_+ n_+ - q_- n_-) \quad (16)$$

where it has been recalled that $s_+ = -s_- = 1$. From (10) and (14)

$$\sigma(eq) \rightarrow \sigma_m = \sigma_+ + \sigma_- = s_+^2 q_+ n_+ b_+ + s_-^2 q_- n_- b_- \quad (17)$$

However, atomic charge neutrality requires the material to have no net charge ($\rho_m = 0$) in equilibrium [8], so $\rho_m = 0$ in (16) giving (for the special case $n_+ = n_-$ i.e., $q_+ = q_-$)

$$q_+ n_+ = q_- n_- \equiv q_m n_m \quad (18)$$

On the other hand, these free charges give rise to the conductivity of the material, so

$$\sigma_m = s_+^2 q_m n_m (b_+ + b_-) \quad (19)$$

must be the intrinsic electrical conductivity of the material. This same procedure applies when $n_+ \neq n_-$ (i.e., some liquids, gases), but then $q_+ \neq q_-$, so (19) remains in the form of (17). However, due to the atomic charge neutrality principle, in equilibrium $\rho_m = 0$ will always be true.

Case 2: (Perturb with a single charge species ρ_p)

Next consider what happens when extrinsic charge ρ_p is uniformly placed in the material. For this case (9) and (13) require

$$\rho = \rho_m + \rho_p = \rho_m + s_p q_p n_p. \quad (20)$$

If only interactions (collisions and transport) without reactions (changes in intrinsic ion concentrations) occur between the material and the inserted charge then ρ_m remains fixed. Hence, with the assumption of interaction without reaction $\rho_m = 0$, so (20) becomes

$$\rho = \rho_p = s_p q_p n_p. \quad (21)$$

Furthermore, (10) and (14) require

$$\sigma = \sigma_m + \sigma_p = \sigma_m + s_p^2 q_p n_p b_p = \sigma_m + s_p \rho_p b_p. \quad (22)$$

Clearly, when the perturbation charge ρ_p is placed in the material, it is expected that coulomb repulsion will decay ρ_p away in time. As a result, it is expected that $\rho_p = \rho_p(t)$ which implies $\sigma_p = \sigma_p(t)$, and, therefore, $\sigma = \sigma(t)$ in (22). The conclusion is clear: when charges are added to a material the electrical conductivity of the material is changed; and not until the charges fully decay does the electrical conductivity of the material arrive back at its intrinsic value. As a result, σ in (5) is not independent of time; hence, (7) is not the general solution to (5).

General Charge Decay Equation

As discussed above, it is clear that (21) and (22) are required in (5); so, (5) becomes

$$\frac{\partial \rho_p}{\partial t} + \frac{\sigma_m}{\epsilon} \rho_p + \frac{s_p b_p}{\epsilon} \rho_p^2 = 0. \quad (23)$$

The material time constant can be defined with the intrinsic material conductivity as

$$\tau_m = \frac{\epsilon}{\sigma_m}. \quad (24)$$

The constants in the third term of (23) can be lumped together as

$$\beta = \frac{s_p b_p}{\epsilon}. \quad (25)$$

With the constants (24) and (25) the partial differential equation (5) i.e., (23) becomes

$$\frac{\partial \rho_p}{\partial t} + \frac{\rho_p}{\tau_m} + \beta \rho_p^2 = 0. \quad (26)$$

The solution to (26) is given by [11]

$$\frac{\beta \tau_m \rho_p}{\beta \tau_m \rho_p + 1} = \frac{\beta \tau_m \rho_{p0}}{\beta \tau_m \rho_{p0} + 1} \exp(-t/\tau_m). \quad (27)$$

It is advantageous to define a time constant specified by the initial perturbation charge as

$$\tau_p = (\beta \rho_{p0})^{-1} = \frac{\epsilon}{b_p(s_p \rho_{p0})}. \quad (28)$$

With some rearranging (27) can now be written with the aid of (28) as

$$\rho_p(t) = \frac{\rho_{p0} \exp(-t/\tau_m)}{1 + \frac{\tau_m}{\tau_p} [1 - \exp(-t/\tau_m)]} \quad (29)$$

which is the correct solution to (5) or (23). Because (29) was derived without specifying the state (solid, liquid or gas) and the nature (good conductor, good insulator, etc.) of the material, (29) can be applied to all materials from very good conductors to very poor conductors (i.e., good insulators).

Definition of a Good Conductor

One way to define a good conductor is to require the third term in (26) to be negligible compared to the second term. By this definition the criteria for a good conductor is $\rho_p/\tau_m \gg \beta \rho_p^2$ or $1 \gg \beta \tau_m \rho_p$. However the largest value is at ρ_{p0} , so, the criteria for a good conductor must be $1 \gg \beta \tau_m \rho_{p0}$. With the aid of (28) this criteria becomes $\tau_p \gg \tau_m$. With this good conductor criterion (i.e., restriction) (29) reduces to

$$\rho_p(t) \approx \rho_{p0} \exp(-t/\tau_m), \quad \text{only valid if } \tau_p \gg \tau_m. \quad (30)$$

Equation (30) is similar to (7) except the material's intrinsic conductivity σ_m must be specified since, as shown in this paper, σ is not a constant.

Definition of a Good Insulator

One way to define a good insulator is to require the second term in (26) to be negligible compared to the third term. Hence, for a good insulator $\rho_p/\tau_m \ll \beta\rho_p^2$ or $1 \ll \beta\tau_m\rho_p$ is required. However, this can only be satisfied at the early stage of charge decay, because later, as $t \rightarrow \infty$ so must ρ_p decay to zero, and, therefore, the requirement $1 \ll \beta\tau_m\rho_p$ eventually always fails. At early time, the only time when the requirement can be met, the largest value is at ρ_{p0} ; so, the criteria for a good insulator is $1 \ll \beta\tau_m\rho_{p0}$. With the aid of (28) this criterion for a good insulator becomes $\tau_p \ll \tau_m$.

Comparison to Other "Insulator Laws"

The numerator and denominator of the single species charge decay equation (29) can be multiplied by $\exp(t/\tau_m)$ to yield

$$\rho_p(t) = \frac{\rho_{p0}}{(1 + \frac{\tau_m}{\tau_p}) \exp(t/\tau_m) - \frac{\tau_m}{\tau_p}} \quad (31)$$

Equation (31) has a form somewhat similar to the so-called *modified hyperbolic law* [12] as developed by Vellenga and Klinkenberg [13] in that both equations contain an exponential term and a time-constants-ratio term in their denominators. However, the two equations are quite different in decay character in the range $\tau_m < \sim 10\tau_p$. However, (29), and hence also (31), reduces to the modified hyperbolic law for $\tau_m \gg \tau_p$.

It is interesting to note that, in general, for $t \ll \tau_m$ the exponential is given by $\exp(t/\tau_m) \approx 1 + t/\tau_m$ which when inserted into (31) gives

$$\rho_p(t) \approx \frac{\rho_{p0}}{[1 + \frac{t}{\tau_p}(1 + \frac{\tau_p}{\tau_m})]}; \quad \text{only valid if } t \ll \tau_m. \quad (32)$$

For a very good insulator $\tau_p \ll \tau_m$ and (32) reduces further to

$$\rho_p(t) \approx \frac{\rho_{p0}}{[1 + \frac{t}{\tau_p}]}; \quad \text{only valid if } \tau_p \ll \tau_m, \text{ and only if } t \ll \tau_m. \quad (33)$$

Although (33) is the *hyperbolic law* of Bustin, et al. [14], the hyperbolic law is often (apparently improperly) assumed to be applicable over all time. As noted above, (33) shows the hyperbolic law is only valid for $t \ll \tau_m$.

Finally, note that $\tau_p^{-1} = \beta\rho_{p0} = [(s_p b_p)/\epsilon]\rho_{p0}$; and, since $\rho_{p0} = s_p q_p n_p$, the product in τ_p always contains s_p^2 . Thus, τ_p is always a positive quantity.

Discussion

Equation (29) derived in this paper is a general equation for charge decay of unipolar ions which have an initial charge density ρ_{p0} and which have been uniformly introduced into any material: conductor or insulator; solid, liquid or gas. It is assumed that the material media into which the charge is introduced can be classified as a simple material; i.e., the material's electrical properties must be isotropic, linear and homogeneous. In the derivation of (29) it is assumed that the inserted charge ρ_p only interacts with the material through collisions and transport and not through reactions which can change the intrinsic ion concentrations of the material. In (29) the intrinsic material electrical relaxation time constant τ_m is given by (24) where ϵ is the material's permittivity and where σ_m is the intrinsic electrical conductivity of the material. The time constant ratio in (29) is

$$\frac{\tau_m}{\tau_p} = \frac{b_p s_p \rho_{p0}}{\sigma_m} \quad (34)$$

where b_p (always a positive quantity) is the electrical mobility of the unipolar ions of initial volume charge density ρ_{p0} and s_p is the sign of the charge (either +1 or -1) of the unipolar ions. The initial charge density of the unipolar ions in the material is

$$\rho_{p0} = s_p q_p n_{p0} \quad (35)$$

and depends on the number of ions per unit volume n_{p0} , and the magnitude of the charge q_p of the ions where $q_p = Z_p q_0$ and where $q_0 = 1.6 \times 10^{-19}$ C is the fundamental unit of charge and Z_p is the number of fundamental units. Note that $s_p \rho_{p0}$ is always positive.

The definition and distinction between a good conductor and a good insulator can be defined using the two electrical relaxation times given in (24) and (28). Namely, the criterion for a good conductor is $\tau_p \gg \tau_m$ and for a good insulator is $\tau_p \ll \tau_m$. Since the mobilities are often of the same order of magnitude in a given material, these definitions also define a good conductor as $n_m \gg n_p$ and a good insulator as $n_m \ll n_p$. These definitions and (29) are useful in specifying the type of charge decay (effectively exponential or early time hyperbolic). Of course, the basic application definition of a good conductor still remains $\tau_m \ll \tau_{use}$ and for a good insulator $\tau_m \gg \tau_{use}$ where τ_{use} is the use time for the process, investigation, operation, etc.

Summary

A general equation (29) has been developed which describes the time decay for a uniform distribution of unipolar charge inserted into any simple material (conductor or insulator; solid, liquid or gas) with an initial charge density ρ_{p0} . The relaxation is controlled by both the intrinsic properties (conductivity σ_m and permittivity ϵ) of the material, and the extrinsic properties of the inserted charge, i.e., the initial volume charge density ρ_{p0} and the mobility b_p of the inserted charge. In (29) the charge density ρ_p is assumed not to chemically react with the material (σ_m invariant to ρ_p) and decays in time t depending on the

initial charge density ρ_{p0} defined by (35), on the material's intrinsic relaxation time constant τ_m defined by (24), and on the perturbation time constant τ_p defined by (28).

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